**Course Title:** Geometry **Full marks:** 100

**Course number:** Math. Ed. 422 **Pass marks:** 35

**Nature of course:** Theory **Period per weeks:** 6

**Level: Bachelor Degree Total Period:**150

### 1. Course Description

This course is designed to provide broader and deeper understanding of different ways of geometric thinking required to prospective secondary school mathematics teachers. It consists of two parts. The first part of the course (Euclidean geometry)) provides basic and fundamental concepts of Euclidean geometry for prospective secondary school mathematics teachers. It deals with foundational elements of geometry as a mathematical system focusing on Euclidean Geometry as a formal system.The focus of the contents are on notion of congruence, parallelism, similarity, convexity, and area and volume. The second part aims to introduce geometries in relation to foundational properties considering critically in the development of new form of geometries. This part deals different formal ways to the study of geometric system such as, the axiomatic systems of Euclidean geometry, neutral geometry, transformational geometry, non-Euclidean geometry, projective geometry and topology.

### 2. General Objectives

The general objectives of this course are as follows:

1. To acquaint the students with foundational elements of geometric systems to construct *foundation of geometry*
2. To enable the students to define the notion of parallelism, congruence, similarity, convexity, and areas from point-set principles and apply these properties in writing proofs.
3. To enable the students to acquire the principles and skills of basic geometric constructions
4. To enable students to characterize the properties of plane and solid convex figures and use these properties in defining plane and solid convex figures.
5. To develop the notion of the area and volume (axiomatic principles) for the derivation of the areas of rectilinear figures and volumes of regular geometrical bodies (regular shapes).
6. To familiarize the students with brief review of historical development of axiomatic systems and together with the major building blocks of geometry.
7. To acquaint the students with the content of neutral geometry and extend it to consider Euclidean perspective as one to deal with geometric properties.
8. To acquaint the students with transformational geometry in which geometric properties are preserved through transformations.
9. To enable the students in understanding the development of non-Euclidean geometry and establishing their properties.
10. To make the students familiar with projective geometry as the generalized form of other geometries (such as, Euclidean, non-Euclidean and transformational).
11. To acquaint the students with topological properties as general type of geometric properties that remain even after deformations.

**3. Specific Objectives and Contents**

|  |  |  |
| --- | --- | --- |
| **Unit** | **Specific of Objectives** | **Contents** |
| I | * Explain the foundation properties of Euclidean geometry. * Differentiate between direct and indirect proofs with examples and examine their logical basis to establish consistency in mathematics. * Describe basic properties of sets in a line, rays and angles, polygons and circles. | **Unit 1: Some foundational Elements of Geometry** 5    1.1 Foundation properties  1.2 Consistency and indirect proof  1.3 Basic properties of sets in a line,  1.4 Basic properties of Rays and Angles.  1.5 Polygons and circles |
| II | * Explain notion and conditions of congruence. * Prove theorems on congruence of triangles and polygons * Solve problems on congruence of triangles and use them to solve others | **Unit II: Congruence**  7  2.1 The notion of congruence  2.2 Congruence condition  2.3 Some theorem on congruence of triangles and polygons  2.4 Problems related to congruence properties |
| III | * Define parallel lines and skew lines * Define Euclid's parallel axiom * Explain conditions of parallelism * Relate properties of parallelism to characterize the properties of quadrilaterals, triangles and circles | **Unit III: Parallelism 7**  3.1 Parallel lines and skew lines  3.2 Axiom of parallelism  3.3 Some theorems on parallelism  3.4 Quadrilateral and its properties  3.5 Properties of triangles and circles |
| IV | * Identify the existence of similar figures as the implication of parallel axiom * State the significance of Parallel Proportion Problem(PPP). * State conditions of similarity Prove theorems on similarity of triangles and polygons * Prove Pythagoras theorem and its converse. | **Unit IV: Similarities 8**  4.1 Implication of the parallel axiom in the existence of similar figures  4.2 Parallel proportion problem  4.3 Conditions of similarities  4.4 Some theorems on similarity of triangles and polygons  4.5 Pythagoras theorem and its converse.  4.6 Problems related to similarity properties |
| V | * Illustrate the concept of, convex set, planar set, nonlinear set, bounded set and convex set * Prove theorems on plane convex figures * Explain convex solids, various concepts related to convex solid, and its properties * Prove theorem on convex solids * Define solid polyhedron, regular polyhedron, prism, pyramid, cylinder and cone and draw appropriate diagrams | **Unit V: Convexity 6**  5.1 Plane convex figures: Bounded planar set, boundary *point*, interior point and plane, supporting line, convex curve  5.2 Theorems on convex set  5.3 Convex solids and its properties, supporting plane  5.4 Theorems on convex solids  5.5 Solid polyhedron: Regular polyhedron, prism, pyramid, cylinder and cone, existence of polyhedrons and five regular polyhedron |
| VI | * State axioms of area and volume State and prove theorems on the area of a right triangle, general triangle and quadrilateral, polygons * Solve the problems related to areas of polygon * Explain the Cavalieri's principle * Prove theorems related surface area and volume of prism, cone, pyramid and sphere * Solve the problem on area and volume | **Unit VI: Area and Volume 10**  6.1 **Area:** Area axiom, triangle, quadrilateral, polygons. apothems, area of circles, cone, parallelepiped, surface area of sphere,  6.2 Selected Problems on calculating area of polygons.  6.3 Fundamental theorems of area of triangle, quadrilateral and polygon  **6.4 Volume of solid:** Volume axiom, Cavalleri's principle, volume of prism, pyramids, cone and volume sphere,  6.5 Selected problems on computing areas and volumes |

**Modern Geometry**

|  |  |  |
| --- | --- | --- |
|  |  |  |
| VII | * Sketch the brief historical development of geometry * Describe axiomatic system and its properties. * State and interpret incidence axioms * Prove existence of incidence properties in finite geometries | **Unit VII: Axiomatic System**  **7**  7.1 Historical background  7.2 Axiomatic systems and their properties  7.3 Incidence geometry: Finite geometries (Four points geometry, Fano's geometry, Young's Geometry) |
| VIII | * Sketch the development of Euclidean geometry, Euclid's Elements, * Explain logical shortcoming of Euclid's Elements * Explain various attempts to prove Euclid's postulate. * State and explain Hillbert axioms, Birkhoff's model and SMSG postulates for Euclidean geometry | **Unit VIII: Euclidean Geometry in Modern Form (13**)  8.1 Euclid's geometry: Its development, Euclid Elements, Logical shortcoming of Euclid's Elements, Euclid's Fifth Postulate, and its consequence, attempt to prove fifth postulate.  8.2 Modern Euclidean Geometries- Hilbert's model for Euclidean geometry. Birkhoff's model for Euclidean geometry, SMSG model of Euclidean geometry |
| IX | * Describe preliminary notion of neutral geometry * Prove theorem related to Saccheri –Legendre and Lambert quadrilaterals | **Unit 9: Neutral Geometry** (14)  9.1 Preliminary notions of Neutral Geometry  9.2 The Saccheri-Legendre theorem,  9.3 The Search for a rectangle: Lambert quadrilateral |
| X | * State the Euclid's parallel postulate and examine its implication in establishing the nature of geometric relations in Euclidean geometry * Explain Euclidean results concerning circles, triangles, | **Unit 10: Euclidean Geometry and its Application in the Plane (6)**  10.1 The parallel postulates and its implications  10.2 Euclidean results concerning circles, Nine point circles  10.3 Euclidean results concerning triangles, Theorem of Menelaus and Ceva |
| XI | * Explain isometric transformations, non-isometric and inversion transformation and their related concepts * Prove the theorems related to the above transformations * Solve the problems related to the above transformation | **Unit 11: Transformational Geometry (18)**  **11,1 Isometric transformations:** Reflection, Translation, Half-turn, Rotation, Glide reflection and their equations in analytic and in matrix form.  **11.2.**Non**-isomeric transformation.** Dilation, Enlargement and reduction, Stretch, Shear.  **11.3 Inversion transformations:** Inverse point, feature of inversion, geometric construction of inverse points. Inverse of a line, a circles and a curve and related equations |
| XII | * Explain the development of non-Euclidean geometry. * Describe the angle of parallelism * Explain development of hyperbolic geometry and its related results. * Explain Elliptic geometry and its related results. * Compare different type of geometry | **Unit 12: Non-Euclidean Geometry 20**  12.1 Development of Non Euclidean Geometry  12.2 The angle of parallelism  12.3 The hyperbolic geometry- Model of hyperbolic geometry, hyperbolic parallel postulates, some results in hyperbolic geometry  12.4 Hyperbolic results concerning polygons  12.5 Elliptic geometry- Model of elliptic geometry, Elliptic parallel postulates, Postulates of elliptic geometry, some results in elliptic geometry  12.6 Comparison of three geometries |
|  | * Explain fundamental concepts of projective geometry and projective properties * Define real projective plane * Prove some elementary properties of projective plane * Discuss the principle of duality in projective geometry * Prove the theorem of Desgargues * Explain projective transformation | **Unit 13: Projective Geometry (15)**  13.1 Fundamental concepts of projective geometry  13.2 Axioms of the Real projective plane- projective properties  13.3 Duality  13.4 Perspectivity  13.5 The Theorem of Desargues  13.6 Projective transformations |
|  | * Discuss the concepts and terminology associated with network * Prove the result of Euler's discovery about networks * Explain Koenisberg Bridge Problem * Describe Polyhedra - Euler's Formula for Polyhedra * Classify surfaces topologically | **Unit 14: Topology (12)**  14.1 Topological transformation  14.2 Networks and their properties-Euler's discovery about networks  14.3 Koenisberg Bridge Problem  14.4 Polyhedra- Regular polyhedra, Euler formula for regular polyhedra dual of polyhedra, and their properties  14.5 Toplogical surface: Torus, genus of a surcae, Euler's characteristic of a surface, moebius strip, Kleins bottle, Orientiability of surfaces |

**Instructional Techniques**

Because of the theoretical as well as concept oriented nature of the course required to prepare sound background for prospective teachers in geometry, teacher-centered instructional techniques as well as techniques based on problem solving, presentation and group discussions will be main instructional techniques. Depending on the nature of the teaching items, the following techniques/methods will be used as general instructional techniques separately or in elective form.

* 1. **General Instructional Techniques**
* Expository techniques followed by Problem Solving
* Discussion, Demonstration and Inquiry
* Presentation and group discussion
* Eclective techniques
  1. **Specific Instructional Techniques**

|  |  |  |
| --- | --- | --- |
| **Unit** | **Chapter** | **Instructional techniques** |
| I | Some foundational Elements of Geometry | Expository, Discussion and presentation |
| II | Congruence of Geometric Figure | Expository, Discussion and presentation |
| III | Parallelism | Expository, Discussion and presentation |
| IV | Similarities | Expository, Discussion and presentation |
| V | Convexity | Expository, Discussion and presentation |
| VI | Area and Volume | Expository, Discussion and presentation |
| VII | Axiomatic System | Expository, Discussion and presentation |
| VIII | Euclidean Geometry in Modern Form | Expository, Discussion and presentation |
| IX | Neutral Geometry | Expository, Discussion and presentation |
| X | Euclidean Geometry and its Application in the Plane | Expository, Discussion and presentation |
| XI | Transformational Geometry | Expository, Discussion and presentation |
| XII | Non-Euclidean Geometry | Expository, Discussion and presentation |
| XII | Projective Geometry | Expository, Discussion and presentation |
| XIV | Topology | Expository, Discussion and presentation |

**5. Evaluation**

The Office of Controller of Examination, Tribhuvan University will conduct the annual examination at end of the year to evaluate students' performance. The questions of theoretical part in the final examination will contains the question from whole course carrying fifty marks. The types, number and marks of the objective and subjective questions that will be asked in final examination by the Office of the Controller of Examination is as follows:

|  |  |  |  |
| --- | --- | --- | --- |
| **Types of questions** | **Total questions** | **Number of questions & their marks** | **Total marks** |
| **Group A:** Multiple choice questions | 20 questions | 1 × 20 marks | 20 |
| **Group B:** Short answer questions | 8 questions with 3 internal choices | 8 × 7 marks | 56 |
| **Group C:** Long answer question | 2 questions (or 1 question) | 2 × 12 Marks | 24 |

**6 Recommended Books:**

Kelly, P. J. & Ladd, N. E.(1986). *Fundamental mathematical structures*. New Delhi : Eurasia Publishing House(P) LTD (Original publication: Scott, Foresman and Company, USA).

Wallace, C. E. & West, S. E. (1998). *Roads to geometry****.*** (Second Edition). USA: Prentice Hall. **(For units VII to XIV)**

**References**

Pandey, U. N. (2012), *Modern geometry*, Kathmandu: Vidyarthi Prakashan Pvt. Ltd. (Chapters I-VIII)

Pandit, R. P. & Pathak, B. R. (2009). *Fundamentals of geometry****.*** Kathmandu: Indira Pandit. **(For units 1-6)**

Pandit, R. P. (2016). Fundamentals *of geometry.* Kathmandu: Indira Pandit. **(For units 7 to 14)**

Pandit, R. P. (2008). *Elementary modern mathematics.* Vol. 1-2, combined, Kathmandu: Indira Pandit. **(**For **units 7 to 14)**